

# Regge Amplitudes and Final State Phases in the Decays $B \longrightarrow D\pi$

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## Abstract

The scattering amplitude of  $D\pi$  at the energy of the  $B$  mass can be calculated using Regge theory. Recent papers have used this to calculate the final state strong phases in the decays  $B \longrightarrow D\pi$ . It is argued that while the Regge amplitude can yield an absorption correction to the decay rate, it is not useful for determining the strong phase.

A number of recent papers have used a Regge analysis to determine the strong phases in the amplitude of  $B$  decay  $\longrightarrow D\pi$  (1-3). Here we discuss the difficulties with these analyses...

The starting point is the generalized Watson theorem based on CPT and unitarity

$$A_f = \sum_i (S)_{fi}^{\frac{1}{2}} M_i \quad (1)$$

where  $A_f$  is the decay amplitude to state  $f$ ,  $M_i$  is the weak decay amplitude in the absence of final-state scattering and  $S$  is the scattering  $S$  matrix at an energy equal to the  $B$  mass. This scattering is primarily due to strong interactions. The sum is over all states to which  $B$  may decay.

In these papers the sum is truncated at the first term, the "elastic scattering" term. As discussed later, this truncation is completely unjustified, but first we pursue this approximation, yielding  $A = (S)^{\frac{1}{2}} M$  where we have omitted the subscript  $f$  so that  $S = S_{ff}$  and we need only consider  $s$ -wave scattering.  $S$  is related to the scattering amplitude  $F$  by

$$S = 1 + 2iF \quad (2)$$

For the case of  $B \longrightarrow D\pi$  there are two final states  $D^+\pi^-$  and  $D^0\pi^0$ ; however using the isospin analysis we can consider separately the  $I = \frac{3}{2}$  and  $I = \frac{1}{2}$  states as the single state  $f$ .

For the case of truly elastic scattering Eq. (2) gives

$$S = 1 + 2i (\sin \delta e^{i\delta}) = e^{2i\delta} \quad (3)$$

so that the phase of  $F$  gives directly the phase of  $\delta$  of  $S^{\frac{1}{2}}$ , but this is not true in the case of  $B \longrightarrow D\pi$ , where the scattering is primarily inelastic. In Refs. 2 and 3 it seems that they in fact do assume the phase of the Regge amplitude gives the final state phase (see just below Eq 26 of Ref.3), which is clearly wrong. If one only considers the Pomeron trajectory the amplitude  $F$  is purely imaginary so that, from Eq 2,  $S$  is real and there is no strong phase. The Pomeron represents the effect of diffractive scattering that scatters out from  $D\pi$  but not back into it.

In the case that the scattering is not purely elastic the element  $S_{ff} = S$  of the  $S$ -matrix can be written

$$S = \eta e^{2i\delta} \quad (4)$$

It should be noted that putting this into Eq (1) gives a factor  $\eta^{\frac{1}{2}}$  in addition to the strong phase. This may be considered as an "absorption correction" to the simple calculation of  $M$  due to the reduction of  $M$  from rescattering to other states. Such a correction was introduced by Sopkovich [4] in the context of the meson-exchange calculation of a scattering amplitude and applied extensively by Gottfried and Jackson in this context [5].

In the case of  $B$  decays if  $M$  is assumed to be calculated accurately in the absence of strong rescattering one should multiply the result by the square root of  $\eta$ . In practice this is not usually done and presumably it is absorbed in the determination of some of the parameters entering the calculation of  $M$ . If the Pomeron dominates the inelastic scattering then using it in Eq.2 may give a good value of  $\eta$  even though it is essentially irrelevant for determining the strong phase.

In the case of eq(4) the phase  $\phi$  of  $F$  is related to the phase  $\delta$  of  $S$  by

$$\begin{aligned} \tan \phi &= \tan \delta + A(\tan \delta + \cot \delta) \\ A &= \frac{(1 - \eta)}{2\eta} \end{aligned} \quad (5)$$

Fayazuddin [6] uses the correct relation between the phase of the  $S$  matrix and the phase of the Regge amplitude as illustrated in Table 3 of that paper. In the examples shown the main effect of the Regge amplitude is to give a value of  $\eta$  around 0.7 while the phase delta is quite small.

In Ref (1) Fayazuddin applies this to the decay  $B \longrightarrow D\pi$  with results that appear to agree with experiment. However there is still the serious problem of

the truncation of Eq. (1). In the Regge analysis the non-zero phase of  $\delta$  arises first via  $\rho$  exchange which gives a real contribution to the elastic scattering amplitude  $F$ .

However  $\rho$  exchange also gives  $D^*\pi \longrightarrow D\pi$  scattering and since  $M$  for  $B \longrightarrow D^*\pi$  is much the same as  $M$  for  $D\pi$  this should double the phase at least in the small phase limit. There also are contributions from the  $D^*\rho$  state although this requires  $\pi$  exchange.

In conclusion the  $D\pi$  cross-section consists of highly inelastic scattering such as that described by the Pomeron which primarily determines the parameter  $\eta$  in the  $S$  matrix. There is also two body to two body scattering such as that described by  $\rho$  exchange. This includes the elastic scattering which contributes to the strong phase  $\delta$  but also scattering into the  $D\pi$  state from other final states which may be least important as the elastic in determining  $\delta$ .

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## References

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